## *Chapter 2*

### *II. Eleven Pitch-Class Systems: Their Relevance and Application to Works in the Literature*

#### *A. The Bach Inventions and WTC I*

We begin this part of the discussion with a simple observation: several of Bach's Two-Part Inventions unfold only eleven pitch classes and the missing pitch class is invariably the same in each case. That is, if one catalogs all the notes used by Bach to write the Two-Part Invention in C major, only eleven are found; the notes of the C major scale, plus C♯, F♯, G♯, and B♭. The note E♭ and its enharmonic, D♯, is absent. In each of the five inventions in C, D, E♭, F and B♭, basically short, diatonic, single-issue compositions, the same eleven notes are used, in the moveable-*do* sense. In the complete set of fifteen inventions, eight are in major keys. (The minor mode, and the special considerations surrounding minor mode in reference to this theory, will be discussed later.) Of those eight, five of them have only eleven pitches and the same pitch class, pc 3, is missing from each of those five. The question may therefore be raised: Does this indicate Bach's sensitivity to an obscure chromatic issue, or is it just coincidental?1 Of the

[1]This is not the first time that questions about the supremacy of eleven notes have appeared in the theory world. Such observations have been made by theorists and composers such as Edward Lowinksy in reference to Renaissance

twelve available pitch classes in C major, pc 3, spelled either as D♯ or E ♭, is the most difficult to unfold in a simple diatonic context without raising various chromatic voice-leading issues. One could argue that Bach omits pc 3 in these five instances, perhaps, to avoid even a very local tonicization of iii, where pc 3 would be spelled D♯, or even a short excursion into the minor mode where pc 3 would be spelled E ♭. However, we believe such potential explanations to be inadequate. Instances of motions to iii within the major mode abound in Baroque compositions at least since the publications of Corelli's opp. 1—4 in the late 1690s. In particular, Baroque concertos often use iii as the penultimate goal before the tonic return. But these pieces are extensive and use the motion to iii to achieve harmonic climax, consequently necessitating "modulations" from one eleven pitch-class system to another (this will be discussed fully in Chapter 3). One could argue, then, that Bach's decision to avoid the entry of a iii triad or to avoid

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motets (see his *Secret Chromatic Art in the Netherlands Motet* [New York: Columbia University Press, 1946]:100, *n34*). Composer Henry Weinberg, professor emeritus at the Aaron Copland School of Music, CUNY, has often observed in his lectures and seminars, that Orlando di Lasso consistently unfolds eleven pitch classes in his motets and madrigals. Henry Burnett has gone a step further by noting in his "A New Theory of Hexachord Modulation in the Late 16th and Early 17th Centuries," that the vast majority of compositions in Monteverdi's madrigal collections 1) unfolds eleven notes and that sectional divisions of longer madrigals separate eleven-note gamuts, 2) at points of modulation, different eleven-note gamuts are unfolded, and 3) in each eleven-note gamut, the same pitch class, pc 3, is missing. This will be further discussed in Chapter 3.

the parallel minor could have been conditioned upon his purposeful avoidance of pc 3 within the context of a short composition.

Sensitivity to an eleven pitch-class issue, with special attention paid to pc 3, continued way past the late Renaissance and the Baroque eras. For example, the slow introduction to Beethoven’s Symphony no. 1 in C major uses only eleven pitch classes, the only one missing is pc 3; by the time it finally enters, we are already in the bridge passage of the exposition. On the other end of the compositional spectrum, the opening period (mm. 1-11) of Schoenberg’s Op. 11, no. 1, uses only eleven pitches and, consistent with its “key signature” of no sharps or flats, the missing pitch is E♭ or D♯ (this will be discussed in Chapter 7). Interestingly, E♭ emerges in a significant way in the following two periods, just before the central section.

Of course, the number of pitch classes a composer chooses to use will vary from one composition to the next depending on length, circumstance and complexity; pieces may draw upon less than eleven pitch classes just as they may contain the total aggregate. Bach’s other three major-mode inventions from this same collection vary in the total number of pitch classes they contain. For example, the Invention in G major uses only nine notes, the seven diatonic ones plus F♮ and C♯. The pitches B♭ or A♯, which would be the enharmonic variants of pc 3 in a 1♯ system (where G is the root and A♯/B♭ is the missing pitch), are not used. The designation “1♯ system” refers to a field of eleven notes where G is the root and where A♯/B♭ is the missing pitch.

In the Invention in A major, pc 3 is present, spelled only as B♯. Each time that B♯, the system-shift motivator, is present (mm. 10–12), it occurs as a localized leading tone, part of an applied chord to C♯ minor (iii), first as $V^7$/iii and afterwards as $VII^{o7}$/iii. The second chordal form, the diminished seventh chord, particularly interests us here because the seventh of the chord, A♮ , is present each time, just after pc 3 enters, effectively preventing the root system of 3♯s from shifting up to a 6♯ system.

To clarify, in a 3♯ system, the system-consonant tritone is A—D♯. The system-dissonant tritone, F♯—B♯, includes the missing pitch of the 3♯ system. Therefore, we could say that the use of the missing pitch, B♯, functioning as a system-shift motivator, propels us three notches clockwise along the circle of fifths into a system whose tritone F♯—B♯ is system consonant, and where F♯ is the root of a new eleven-note system. As a result, F♯—B♯ becomes the system-consonant tritone of the new 6♯ system. The missing pitch, pc 3, of the 6♯ system, is A♮ (a minor third above F♯), the root of the original 3♯ tonic system. (The other possibility would be Gx, the enharmonic of A♮ , a note which does not appear in this invention.) We contend, therefore, that the interaction of system-shift motivators between B♯ and A♮ in A major has both a dynamic disruptive *and* stabilizing effect. Thus, in any major-mode tonality, a diminished seventh chord that contains pc 3 as one of it members (spelled either as a minor third or an augmented second above the tonic), simultaneously contains both system-shift motivators: the missing pitch of the tonic system and the missing pitch of the complementary dissonant system. The two pcs effectively "cancel each other out," preventing a system modulation. "System modulation" or "system shift" refers to the motion, either "real" or "implied," from one eleven pitch-class field to another occasioned by the introduction of pc 3, the missing pitch of the prevailing eleven pitch-class field. A real modulation will remain in the new eleven pitch-class field whereas an implied modulation will immediately introduce the "correcting" missing pitch needed to return to the tonic system. The B♯ diminished seventh chord referred to in the A major invention would therefore be said to imply a shift of system but without realizing such a shift.

The relationship between systems and harmonic areas needs to be briefly addressed here. The modulation from a 3♯ to a 6♯ system in the A major invention is locally articulated by the prolongation of, first, A major and, afterwards, by C♯ minor. Although C♯ minor as a tonic entity would have 4 sharps in its key signature, here we find that the harmonic area around C♯ minor is subsumed under a briefly expressed 6♯ system; that is., C♯ minor as a localized harmonic area is a subset of a 6♯ system. As previously stated, it is important not to confuse the key signature of a diatonic prolongation (had it really been a tonic) with its system; only the key signature of the background tonic of a composition and that of its tonic system will be identical. Similarly in tonal voice-leading analysis, a brief -- or even a lengthy -- excursion into a non-tonic harmonic area does not usually signal a motion away from the background tonic sufficient to tonally displace it.

The final invention to be examined is the one in E major, the only one of the group of fifteen that Bach organized in binary form. This invention is an excellent example of the dramatic interplay of opposing pitch classes of competing consonant and dissonant systems. Again, like the A major invention, the E major invention uses pc 3 spelled exclusively as an augmented second above the tonic. In E major, a 4♯ system with E as its root, the missing pitch may be spelled either as G♮ or Fx; here, however, Bach consistently uses the Fx spelling. Therefore, each use of Fx, as a system-shift motivator, will propel us from the tonic 4♯ system to its complementary 7♯ system (Fx divides the C♯ octave symmetrically), and, consequently, the tonic, E, will bring us back into the tonic system again.

Here, the missing pitch does not appear until the first part of the B section, after the first double bar, during a tonicization of G♯ minor (iii in E major) that succeeds a cadence on the dominant at the end of the A section (Ex. 1). An Fx is presented as a leading tone in this area and, in m. 29, brings us into a 7♯ system; when E♮ finally enters uncontested in m.32 (the effect of the E♮ in m.30 is immediately negated by the Fxs surrounding it), it momentarily restores the tonic 4♯ system. However, between mm. 37 and 43, a tug-o'-war between Fx and E♮ is finally settled in favor of E♮ and the conclusive return to the tonic 4♯ system in m. 43 coincides with the uninterrupted final prolongation of tonic harmony. In fact, the cadence in G♯ minor in mm. 41– 42 is directly followed by a return to the opening measures of the invention in invertible counterpoint; there is no B major dominant, nor is there, it seems, the need for any, to smooth over the transition from G♯ minor to E major: the return of E♮ is sufficient to restore both the tonic system and the tonic key without any further harmonic elaboration.


**EXAMPLE 2.1: Bach, Invention in E major, mm. 21-42**

One can only speculate as to Bach's motivation for exclusively using the "sharp" spellings of pc 3 in the major-mode inventions that employ all twelve notes. However, two fundamental points need to be stated here: when a composition presents only eleven pitch classes, it will invariably be pc 3 that is absent; second, when that twelfth note appears, its deployment will initiate a chain of events that will figure prominently in the development of that composition.

We postulate, then, that the introduction of pc 3 creates a unique kind of chromatic dissonance which effects a disruption of the tonal fabric in the prevailing eleven-note pitch field. However, it is the crucial interaction between the system-consonant tritone and its system-dissonant complementary tritone that gives rise to a special affiliation between system-shift motivators – pc 3 of the tonic system and pc 3 of the complementary systems. In a "0" system, such a partnership of motivators would exist between the notes E♭ and F♯ (where the presence of E♭, pc 3 of the "0" system, would create an affiliation with F♯, pc 3 of the 3♭ system) or between the notes D♯ and C (D♯, pc 3 of the "0" system, would interact with C♮, pc 3 of the 3♯ system). It should be noted, too, that the intervallic relationship between these pairs of notes is either a *minor third* or an enharmonically respelled *minor third*. The two tritone pairs form a larger network of minor third relationships that *operates beneath the surface level of the tonic unfolding*; an equalizing of the interaction between the tonic unfolding on the one hand, and the tritone systems operating underneath on the other, becomes increasingly relevant toward the end of the nineteenth century and into the twentieth century.

In compositions where all twelve pitch-classes are present, pc 3 invariably appears in ways that are noticeably distinctive. We have examined some examples where pc 3 is written as an augmented second above the tonic. Now we will examine the first prelude in C major from Bach's *Well-Tempered Clavier* (Ex. 2), in which pc 3 is consistently spelled as a minor third above the tonic C. In m. 19, where the opening measure reaches its octave transposition point, Bach has already unfolded eleven notes and pc 3 has not yet entered. As the piece continues, however, E ♭ finally enters in m. 22, but with the simultaneous appearance of F♯. As the music continues during the dominant pedal, E ♭ enters one more time, again in conjunction with F♯. Significantly, the C major fugue that follows unfolds only eleven notes and, as you might guess, the missing note is pc 3. Incidentally, the C major prelude and fugue from Book II follows the same system organization as Book I: the prelude only uses pc 3 in very close proximity to F♯, while the fugue has only eleven pitches, omitting pc 3.

**EXAMPLE 2.2: Bach, Prelude in C major, WTC 1**

Bach’s use of E ♭ raises the possibility of a modulation from a “0” system to a 3 ♭ system. However, E ♭ appears with F♯, the missing pitch of a 3 ♭ system. The simultaneous deployment of both system-shift motivators, E ♭ and F♯, effectively prevents any system modulation by presenting the missing pitch of the tonic system along side the missing pitch of its complementary system. By having both E ♭ and F♯ in the same diminished seventh chord, Bach simultaneously destabilizes and "restabilizes" the musical fabric. Without F♯, the potential for motion into the tonic minor would be far too strong, disrupting the purity of the tonic major, the presentation of which is the primary concern of the opening prelude. The F♯ thus effectively prevents us from hearing a stark major/parallel minor here. In addition, Bach employs another strategy to highlight his missing pitch. In the 35-measure prelude, the E ♭ appears in m. 22 which is approximately 62% through the music. The 22:35 ratio is the closest one can come to the Golden Section proportion where there is only one chord per measure. But there is more: the next entry of E ♭ is in measure 28, which is the fifth measure of an eight-measure pedal point; 5:8 is also a Golden Section proportion.

The tritone symmetries that govern symmetrical divisions within the major mode can be condensed into a single illustration. Figure 2.2 illustrates “The Tonic System Matrix of Symmetrical Axes” (hereafter referred to as “The Tonic System Matrix”) of a theoretical composition in either C major or A minor, since both keys share the same key signature, and therefore the same system matrix. Compare Figure 2.2 with Figure 2.3, “The Circle of Fifths”. In Figure 2.2, if one starts with the note C (representing a C eleven pitch-class system), and proceeds to the right, the introduction of E ♭, borrowed from the parallel minor, moves the “0” system to a 3 ♭ system. Going up diagonally up to the left, F♯ (the tritone divider of the C octave and, of course, the missing pitch of a 3 ♭ system) is required to return us to the original “0” system. The upper member of any given tritone must be spelled as a sharp (or a natural in a flat system) since the note written as a flat does not appear within the chromatic octave of the eleven pitch-class system. That is, within the C chromatic octave, G ♭ could not functionally replace F♯ since G ♭ cannot act as a leading tone anywhere on the usable circle of fifths (that is, only allowing for a usable circle of fifths between 7 flats and 7 sharps). In order for G ♭ to appear, its presence would have to be prepared at least by E ♭, the missing pitch of the “0” system.

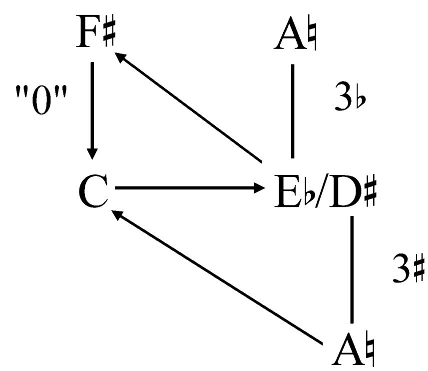


**FIGURE 2.2: The Tonic System Matrix of Symmetrical Axes**

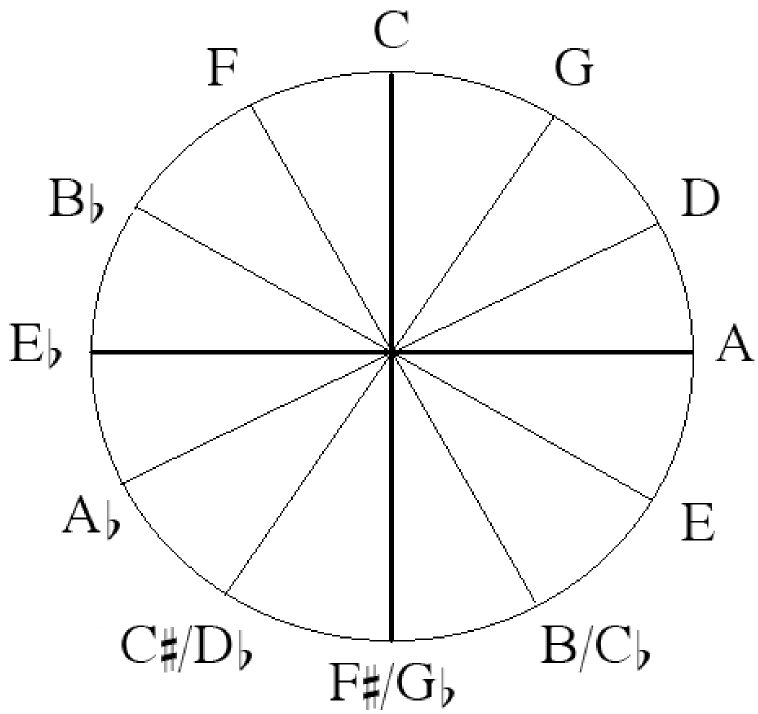


**FIGURE 2.3: The Circle of Fifths**

To reiterate, if pc 3 is spelled as D♯ in a “0” system, a set of similar, but "mirror-image", assumptions can be made. Since D♯ is the tritone divider of the A octave, its use in a “0” system will signify a move up three notches along the circle of fifths into a 3♯ eleven pitch-class system with A as its root. The complementary dissonant tritone system to the A system would then be C—F♯, C♮ (or B♯) being the missing pitch of the A 3♯ system. However, in order to return to the original “0” system the missing pitch of the 3♯ system must be spelled C♮ (the minor third of the A 3♯ system), and not B♯ (the augmented second) since the modulation requires a move down three systems to “0”, something that can only be accomplished if the missing pitch is spelled as a minor third. On the other hand, if B♯ were introduced instead of C, B♯ would indicate (or, rather, motivate) a modulation another three systems up (clockwise) from a 3♯ system to that of a 6♯ system where B♯ symmetrically divides the F♯ octave.

We therefore postulate that the missing pitch of any tritone system, when spelled as a sharp, acts as the octave divider of a system whose root is a tritone below that sharp pitch class. However, if the missing pitch is spelled as a flat, that flat pitch class itself indicates its function as the root of its own tritone system. In addition to this, one may also consider, in a "0" system, that the missing pitch spelled E♭ is the root of the complementary 3♭ system; if the missing pitch is spelled D♯ (a pitch not on the usable circle of fifths), then it becomes the tritone divider of the 3♯ system.

Notice in the circle of fifths (Figure 2.3), that relationships between systems may also be described as relationships between complementary tritones at right angles to one another. Using this diagram, one might quite accurately refer to a "0" system as *a C tritone system*; that is, an eleven pitch-class system defined by its system-consonant C—F♯ tritone. (The terms "system" and "tritone system" have the same meaning, since both refer to eleven pitch-class areas determined by a system-consonant tritone, and may therefore be used interchangeably.) Similarly, the introduction of E♭ into the C tritone system would define a system delineated by the E♭ complementary tritone, E♭—A♮. Notice, too, that in order to get from one system to another, one will constantly move back and forth between the vertical and horizontal axes, or from the horizontal to the vertical.

These diagrams show that the use of pc 3 in any eleven-note field will invoke a system modulation along the circle of fifths by a minor third either in a clockwise direction or in a counterclockwise direction by ninety degrees depending upon the enharmonic spelling of the system's missing pitch; and, that the enharmonic spelling of pc 3 is absolutely crucial to an understanding of the direction in which one moves around the circle. As a consequence, this theory is incompatible with any theory that espouses enharmonic equivalence. In Bach's C Major prelude above, for example, the simultaneous employment of E ♭ and F♯ was necessary to preserve the "0" system. Had F♯ not been used to counteract the system-modulatory potential of E ♭, and G ♭ had been used instead, that would have brought the systems further counterclockwise from a 3 ♭ system into a 6 ♭ system, opening up a Pandora's box of chromatic possibilities far more typical of Chopin than of Bach. Therefore, as discussed above, the concurrent use of E ♭ and F♯ maintains the prevailing tonic system by provoking a system modulation and simultaneously negating it – or in other words, the simultaneous presence of the two system-shift motivators effectively cancels the potential for any system modulation.

Obviously, there are many ways of showing the symmetrical relationships of an eleven pitch-class system graphically. As we have said, the simultaneous presentation of E ♭ and F♯ (or the simultaneous presentation of D♯ and C♮ ) in a "0" system would stabilize the eleven-note environment and prevent it from modulating into either of its two complementary systems. This can more easily be seen in Figure 2.4.

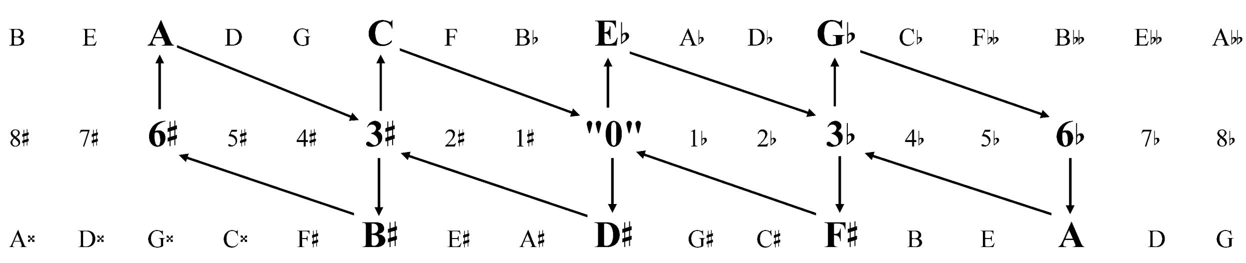


**FIGURE 2.4: Table of Eleven-note Systems From 8 Flats to 8 Sharps**

In this figure, the middle line represents the tonic notes of the major-mode form of 11-pitch class systems from 8 flats to 8 sharps; C is in the middle. Each note of the top line is the missing pitch of the system named directly underneath spelled as a minor third above the tonic of the major-mode form of that system. Therefore, the missing pitch of a 3♯ system may be spelled C. Each note of the bottom line is the missing pitch of the system directly above spelled as an augmented second above the tonic of the major-mode form of that system. In a 3♯ system, the missing pitch may also be spelled B♯.

To determine the directions that 11-note systems move with the use of the each system’s missing pitch, the arrows must be followed in the directions indicated: the use of E ♭ in a “0” system will move the pitch field to 3 ♭s. In the 3 ♭ system, F♯ must be used to return to the “0” system; however, the missing pitch in a 3 ♭ system spelled as G ♭ will continue to move the pitch field further from the original “0” system, now into 6 ♭s. Notice that the motion from one 11-note system to another is consistently three “notches” along the horizontal axes.

The long diagonals between the top and bottom lines are the system-consonant tritones for the 11-pitch class system in the middle line. Therefore, C (on the top line) and F♯ (on the bottom line), indicate the unique system-determining tritone of a “0” system, while E ♭— A is the system-determining tritone of a 3 ♭ system and A—D♯ is the system-determining tritone of a 3♯ system.

One of the tenets of systems theory is that the tonic tritone system of any given movement operates on the deepest level of structure and interacts with more middleground harmonic expansions (that is, expansions involving symmetrically related tritone systems) that constantly play against it. Our thinking here is consistent with harmonic analyses that conceives of the second harmonic area (or second theme group) of a major mode sonata exposition as V of a larger background tonic, not as a modulation that displaces the tonic key of the movement. The authors go a step further: *beneath the tonic key itself is a an even deeper background system of* *minor thirds and augmented seconds comprised of symmetrically related eleven pitch-class tritone systems*. Consequently, we propose that tonality, along with its hierarchal structure, and its division into twenty-four major and minor keys, is ultimately derived from a chromatic gamut of all available pitch classes that is partitioned into diatonic scales of differing whole- and half-step patterns. Thus the chromatic aggregate actually underlies all tonal types and keys.

### *B. The Minor Mode as Re-ordered Root Octave*

It was stated previously that the system analysis of the minor mode required special mention. In this analytical method, the minor mode takes the system of its relative major, just as in the circle of fifths, a minor scale would always be plotted at the same locus as its relative major. Thus, a movement in D minor, such as the second movement from Beethoven's Op. 10 no. 3 sonata, would relate to a 1 ♭ system: both D minor and F major have the same tritone consonant/dissonant conditions that govern a 1 ♭ system and therefore must have the same missing pitch. This relationship of system to mode may help us to understand the minor mode's remarkable degree of tension: its tonic triad is not coordinated with its system's root, and always seeks to move towards its relative major (meaning the root of the key of its tonic system) in order to resolve this conflict. This is why second harmonic areas that are in the relative major in the expositions of minor mode sonata-form compositions seem to have a harmonically stabilizing effect: the transition from the tonic minor to the relative major represents a motion towards the root of the system that is shared by both minor tonic and its relative and, therefore, a motion from instability toward stability.

We would therefore conclude that the minor mode derives from its relative major; that is, the minor mode is a rotation of the pitch classes that form the octave of the relative major: *do, re, me* of the minor thus equates with *la, ti, do* of the relative major. The major mode acts as a gamut of available pitch classes similar to the Medieval/Renaissance gamut from which it ultimately derives. Both gamuts support a number of tonalities, however, whereas the Medieval gamut gave rise to anywhere from eight to twelve different octave species, the tonal gamut (or system, as we chose to call it) supports only two, the major and its relative minor. Likewise, both gamut systems are constructed of major scalar patterns, there being no minor mode equivalent of a major hexachord or scale.

Our view of the minor mode as a reordering of major is not without historical precedent. As evidence, on the title page of *Das wohltemperirtes Clavier*, Book I, Bach describes the content in this manner: “The Well-Tempered Clavier, or preludes and fugues through all the tones and semitones, both as regards the *tertia* major or Ut Re Mi and as concerns the *tertia* minor or Re Mi Fa ...”[2] Again, we see Bach considering the minor mode (“minor mode” itself was terminology that had yet to become popular) as deriving from the gamut of the major hexachord. In his *Treatise on Harmony*, Rameau similarly considers the minor mode conforming to the intervalic structure of the notes within the Re-re octave or the La-la octave in the revised version of the treatise.[3]

In an advertisement by Carl Philipp Emanuel Bach in the *Critische Nachrichten aus dem Reiche der Gelehrsamkeit* of May 7, 1751, the son of the late composer writes about the

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[2]Hans T. David and Arthur Mendel, *The New Bach Reader*, revised and enlarged by Christoph Wolff (New York: W. W. Norton and Co., 1998):97.

[3]Philip Gossett, trans. (New York: Dover Publications, Inc., 1971):263.

upcoming publication of *Die Kunst der Fuge*, in which he describes his father's compositional procedure for this work: "All these manifold fugues are composed upon one and the same principal theme, and in the same key, namely *D minor, or D La Re with the minor third*" [our emphasis].4 One may conclude, then, that the theoretical derivation of the minor from the major continued even after the death of J. S. Bach.

In some minor mode compositions, the composer may intend the transformation of the tonic minor into its parallel tonic major over the course of the entire piece, a motion involving two symmetrically related tritone systems. Haydn, for example, very often followed such a procedure. In the opening of the 1785 Symphony no. 83 in G minor, the first symphony of his Paris set, Haydn immediately begins an interplay between B ♭ and C♯ within the opening melody: G, B ♭, C♯, D. While C♯, the system-shift motivator of the G minor 2 ♭ system, would potentially move us up into a 1♯ system, the simultaneous presence of B ♭ in the violas and cellos prevents any system modulation from occurring. The simultaneous presence of these two pitch classes prevents any system modulation from the prevailing 2 ♭ system throughout most of the first movement's exposition and development. However, by the end of the recapitulation's opening statement, the system-modulatory potential of the missing pitch is realized when, in m. 144, C♯ moves us unequivocally into a 1♯ system in the absence of any further B ♭'s. A couple of measures later, at the beginning of the bridge in m. 146, Haydn accordingly alters the key signature to one sharp and the remainder of the recapitulation, with its absence of the missing pitch — in either of its enharmonic variants — maintains both a G-major tonality and its associated 1♯ system for the rest of the movement. Haydn later duplicates this procedure almost

[4] Ibid.:257.

exactly in the only minor mode symphony of the London set, no. 95 in C minor of 1791. After the recapitulation in the tonic C minor, Haydn, in a manner similar to his earlier G minor symphony, changes key signature to that of the parallel major for the second harmonic area (m. 129, *Maggiore* is indicated in the score at this point). From here to the end, E♭ is entirely absent, allowing F♯ to maintain the "0" system throughout. However, Haydn does offer the enharmonic equivalent, D♯ in place of E♭, balancing the flat-system tendency of the previous minor mode. In order to maintain the "0" system, however, the modulatory potential of D♯ is consistently negated by the simultaneous presence of C♮. Significantly, not only do the first movements of both symphonies end in the opposite major mode system, but both of their finales are entirely in the parallel major confirming the system modulation to the complementary system already accomplished at the end of their respective first movements.

### *C. Further Consequences of System Interactions*

One of the more interesting consequences of system modulations (implied or real) are the foreground dyad conflicts that result and that are developed, or worked out, over the course of an entire composition. In the case of Haydn's Symphony no. 83 in G minor, beside the tug-o'-war between C♯ and B♭ raised early on, a B♭/B♮ dyad conflict in the bridge results from the constant pull towards the parallel major (G major) away from the prevailing tonic minor (G minor). Such pitch-class conflicts, which often become the deepest-level gambits in a composition, form one of the most important developmental processes in music of the eighteenth and nineteenth centuries (and perhaps even the twentieth). The very fact that eleven pitch-class systems comprise tritones organized around minor third cycles naturally raises chromatic pitch classes that are inflections of diatonic ones belonging to the key. Every time a diatonic pitch class is inflected to its chromatic neighbor (either flat or sharp), a conflict arises which eventually must be resolved with either the chromatic inflection returning to its diatonic counterpart, or, as in the case of Symphony no. 83, with the chromatically inflected pitch "winning out", as it were, displacing the initial diatonic pitch class altogether, supported by a true modulation to the alternate system -- here, a 2 ♭ system to a 1♯ system.

But what if "a true modulation to the alternate system" does not happen when, indeed, it is truly expected? The transformation of C minor into C major in Beethoven's Symphony no. 5 is often heralded as one of the most ecstatic artistic expressions of the ultimate triumph of will over fate ever conceived in a musical setting. Yet, at least one commentator has suggested that Beethoven maintains a guarded skepticism about that victory right up to the last note of the last movement.5 Using system analysis to verify the musical content of the transformation, however, we find that Owen Jander may have stumbled upon an idea that has an uncomfortable validity.

The first scheme that offers the potential transformation of C minor into C major occurs at the second harmonic area of the first movement's recapitulation, in m. 303, where the thematic material that had previously been in E ♭ major in the exposition is now transposed to C major. Thus, Beethoven has generated a dyad conflict between E ♭ and E♮ . The F♯'s in m. 296 prepare for this potentiality by moving the tonic 3 ♭ system up to a "0" system; the "0" system is

[5]Owen Jander, "'Let Your Deafness No Longer Be a Secret – Even in Art,': Self-Portraiture and the Third Movement of the C-Minor Symphony," *Beethoven Forum 8* (2000):25-70.

maintained throughout the rest of the recapitulation, and we are led to believe that C major has triumphed over C minor, procedurally similar to Haydn's minor mode symphonies mentioned above. However, the coda, beginning in m. 374 begins a process that undermines such a contention. Both E ♭ and F♯ enter in m. 390, still delaying the return of the 3 ♭ system. However, by m. 400, E ♭ "wins" uncontested because of the absence of F♯. Of course, Beethoven, even during the short excursion into C major never alters the key signature; the return to C minor, and, of course, to a 3 ♭ system, is no surprise.

The opening thematic material of the second movement resurrects the E ♭/E♮ dyad conflict. Harmonically, the positive outbursts of C major later in the movement always succumb to the prevailing A ♭ tonality: E♮ is always displaced by E ♭.

By the end of the C-minor third movement (the scherzo), the tonic 3 ♭ system has been stabilized again after the trio's short excursion into C major. This is the point where we would expect uncontested F♯'s to change the system from the prevailing 3 ♭s to a "0" system; *but this is just what Beethoven avoids*. In m. 324, the timpani ostinato on C begins and the lower strings gradually move from A ♭ to G, and then, in m. 344, to F♯. We would expect that this F♯, or perhaps the one just before it in the violins, would be sufficient to finally maneuver us from a 3 ♭ system back into a "0" system, but it cannot since the E ♭ in the first violins prevents such a system modulation. In fact, each time F♯ occurs now, there is always an E ♭ afterwards to sustain the tonic 3 ♭ system. And, if we take Beethoven's *attacca* at the end of the third movement as an indication of the continued domination of the 3 ♭ system even into the fourth movement, we find ourselves gloriously in C major, with no sharps or flats in the key signature, *and still stuck in a 3 ♭ system!*

The same impasse is reached in the fourth movement as well. Certainly, within the context of a C major tonality, Beethoven, had he wanted, would have had no problem adjusting the prevailing system up from 3 ♭s, thus coordinating the key signature and the system. Just after the opening of the exposition's bridge (m. 26), F♯'s permeate the terrain, so that by the time the second harmonic area enters in m. 44, in G major (with its F♯ leading tone), we are now in the expected "0" system. The continual presence of F♯s in the closing period and coda simply stabilize the "0" system. However, the development turns back to the 3 ♭ system (m. 106) and continues in the flat direction to a 6 ♭ system (m. 109). The A♮ in m. 134 brings us back to 3 ♭s, and the F♯ in m. 143 transfers us up into a "0" system during the development's retransition. But then there is that short, menacing recall of the third movement between the end of the retransition and the opening of the recapitulation; and with it, there is an unequivocal return to the 3 ♭ system since not a single uncontested F♯ is present to successfully return us to a "0" system before the recapitulation — the modulatory potential of each F♯ is canceled each time by E ♭. Therefore, at the recapitulation (m. 207), we just carry on in that fateful 3 ♭ system. The last attempt for the "0" system to displace 3 ♭s occurs in m. 349. At this point, we are in the coda, just before the *più allegro* and *presto*. And again, the promise of F♯, that single note that should have proclaimed the triumph of will over fate, is dashed each time by an omnipresent E ♭. Not a single F♯ enters again before the movement closes. In fact, E ♭'s inability to be successfully displaced by E♮ may explain Beethoven's saturation of the C major surface details with numerous A ♭s and B ♭s even in the *più allegro* and *presto*. Contradicting the final bombast of unrelenting C major harmony, the symphony actually ends in a 3 ♭ system! Beethoven's seeming victory over his fate — his deafness — may not be as convincing as we might be led to believe, with a slight element of doubt encrypted into a hardly noticeable but very present misalignment of diatonic key and eleven-note system.

In his Symphony in C minor, we see Beethoven's potential for optimism always offset by an underlying sense of pessimism related to his worsening deafness and, perhaps, an ever-pervasive cynicism about his future. Our interpretation thus supports Jander, who analyses the movement as a manifestation in sound of Beethoven's attempt to describe his deafness compositionally.

### *D. Major Third Relations Within Diminished Systems*

A question that often arises in systems analysis concerns compositions that unfold major thirds more prominently on the foreground than minor thirds. Beethoven's *Waldstein* sonata, Op. 53, for example, subdivides the exposition of the first movement by major thirds and, consequently, raises some interesting enharmonicism. The sonata's first enharmonic issue occurs with the introduction of pc 3 within the prevailing "0" system. Initially presented as E ♭ in the opening statement (sending the systems down to 3 ♭s), the counterstatement/bridge quickly raises F♯ (bringing us back into the "0" system) and soon introduces the D♯ that effects the modulation to E major, the movement's second harmonic area. The D♯, however, not only signals the arrival of E major, but also shifts the "0" system up to 3♯s. Beethoven's achievement here is striking: by evoking a 3♯ system without reintroducing its missing pitch, C♮ , until the very end of the exposition (thus restoring the original "0" system), he has effectively modulated from one tritone system to another, *the new system displacing the old over a substantial area*.

The 3♯ system remains in effect almost until the end of the exposition. What is revolutionary here is the fact that the new system remains in control and is uncontested for so long.

Unlike diatonic analysis, which would define the second harmonic area in terms of a key signature of four sharps, chromatic system analysis would subsume the E major harmonic area under the prevailing 3♯ A—D♯ tritone system. The transposition of the E major second harmonic area to VI (A major) in the recapitulation, heard within the tonic "0" system, would tend to bolster this interpretation. Returning to the exposition, C♮ is re-introduced in the closing period in m. 70 (against a D♯ which prevents a system modulation),and again in m. 74 (this time without the D♯) at the codetta to effect a return to the original "0" system. This whole exercise in which the 3♯ system is effectively kept from materializing, is necessary, apart from its dramatic intensity, in order to prepare for the repeat of the exposition, which must naturally be in the tonic system. Additionally, ending the exposition in the tonic system creates a neutral pitch field from which Beethoven can now explore the most wide-ranging system modulations in the development section. Again, from the standpoint of system analysis, even a division by major thirds will have an underlying minor third organizational imperative behind it.

Whereas a voice-leading analysis of the *Waldstein*'s first movement would emphasize the role of the E as dividing the space between C and G, and thus relegate the E to a secondary role, system analysis views as significant the harmonic area of E in respect to its role within the symmetrical tritone divisions of a "0" system, with Beethoven playing off the system-consonant C—F♯ tritone against the system-dissonant quality of the A—D♯ tritone. The A—D♯ 3♯ system introduces its own symmetrical properties that are ultimately used to reestablish the tonic "0" system at the end of the exposition. Therefore, another of the many things one can derive from this method of analysis involves an understanding of the system-modulatory potential of enharmonicism.

Sometimes, in music that is decidedly diatonic, the notation of the twelve-note aggregate may involve only twelve spellings. That is, in a simple C major symphony, the use of the chromatics C♯, E♭, F♯, A♭ and B♭ may be sufficient to create a chromatic environment that is capable of modulating out of system, but is also equally capable of returning quickly to the tonic tritone system without intervening system modulations. However, in a more chromatically involved composition, F♯ could be associated with its enharmonic G♭. If pc 3 (again in a "0" system), is spelled as E♭, and there is no F♯ present, this would shift the system down to 3 ♭s. However, if G♭ (instead of F♯) is presented after E♭, the 3♭ system would then continue the motion downward and in a counterclockwise direction along the circle of fifths toward a 6♭ system. In order to restore the "0" system, an A♮ must enter, returning the system to 3 ♭s, and, finally, F♯ must reenter to return the tonic to its original "0" system.

In his earlier works, Beethoven was reasonably content to unfold three symmetrically related systems: the tonic and one on either side of it. For example, the *Cello Sonata* in F major, op. 5 no. 1, uses only the 1♭ system (F), the 4♭ system (A♭) and the 2♯ system (D), according a limited spectrum of enharmonic pairs. However, remaining within the same key and genre, in Brahms's *Cello Sonata* in F major, op. 99, the chromatic texture becomes sufficiently involved to traverse five systems: seven flats (C♭), 4 ♭s (A♭), 1♭ (F, the tonic system), 2♯s (D), and 5♯s (B). In many of Brahms's other works, however, the authors have noticed that he may go through eight or nine system changes before returning to the tonic system. In the Debussy *Cello Sonata*, also in a 1♭ system (and, in this case, in D minor), we find that the first movement uses three systems: 4 ♭s, 1♭ (tonic), and 2♯s; the second movement goes through four systems: 7 ♭s, 4 ♭s, 1 ♭ (of course) and 2♯s; and the third movement uses five systems: 10 ♭s, 7 ♭s, 4 ♭s, 1 ♭ and 2♯s.

One may then surmise, as triadic tonality began gradually to dissolve toward the end of the nineteenth century, what remained was the same eleven pitch-class symmetrical mechanism that had always been beneath the surface of diatonic key-centered tonality, and perhaps modality as well. By the time of Bartók (for example, in his *Music for Strings, Percussion, and Celesta),* the situation had reversed itself to the extent that the composer's triadic sonorities were now completely subsumed under the symmetrical rubrics of unfolded tritone systems.6

### *E. Tonality as Hexachordal Reordering*

Key-centered tonality is often referred to as a hierarchal system that is articulated by progressions of major and minor triads prolonging structurally significant harmonic areas that support a background tonic. The exact nature and relationship of these large-scale harmonic progressions, has not, as yet, been fully explained or explored in the literature other than to say

[6]As an interesting corollary to this discussion, see Ernó Lendvai, *The Workshop of Bartók and Kodály* (Hungary: Editio Musica Budapest, 1983):270-317, 757-762. In his theoretical descriptions of the music of Bartók, Lendvai also indicates his interest in minor-third and tritone relationships. However, Lendvai's starting point is a decidedly Reimannian one where keys related by minor thirds to a tonic, and their tritone implications, are all defined as relatives along symmetrical axes. Thus, according to Lendvai, any member of the minor-third cycle based on the tonic pitch (e.g., C-E♭-F♯-A) may substitute for the tonic itself, a position decidedly at variance with the theory of tritone systems presented here.

that they project a cadential progression on the background level. So long as tonality is heard as nothing more than a progression based on the relationship of tonic to dominant, with the subdominant lying somewhere in between, a full realization of the unique interrelationships of harmonic areas controlled by the background tonic will remain undisclosed.

What we propose is nothing less than a re-definition of tonality itself as something more than just an octave species which supports chords of fixed quality (major and minor). If we accept a tonality as comprising an eleven pitch-class field or gamut from which harmonic areas are derived, we have considerably extended the definition. But what determines the ordering of harmonic areas? Although this concept will be expanded greatly in subsequent chapters, we will introduce the essential points of this issue here.

Part of a potential answer lies in the origins of the tonal system which is ultimately derived from the medieval gamuts of overlapping hexachords.7 The tonality of C major is thus the last step in a long evolutionary process that converted the C *naturalis* gamut, itself allowing eleven pitch classes, to the key of C major. The same could be said for all the other keys: the F *mollis* gamut became F major, the G *durus* gamut became G major, etc). In fact, the C hexachord (the central hexachord of the *naturalis* gamut), which originally controlled the harmonic motions of modally-derived composition, never really disappeared, but simply evolved into the first six notes of the C major scale. Throughout the entire sixteenth and seventeenth centuries, notes of the C hexachord (or the F hexachord, depending on the signature), reordered in fifths, F – C – G – D – A – E, provided most, if not all, of the harmonic motions within a single composition, often in direct contrast to the prevailing modality. However, instead of these fifths supporting a background tonality, they were often related more locally, one fifth progressing to the next, moving both up and down the reordered fifths of the hexachord. (See Chapter 3 for a more detailed discussion of the modal hexachord and its harmonic implications.)

A diagram of the C major hexachord reordered as fifths follows. On either side of the primary hexachord are single and double bars respectively separating tones outside the main hexachord and indicating differing tonal functions. Starting at the left, the B ♭ followed by a single line, represents the added flat seventh degree, and is the only allowable flat in the C major hexachord system (more fully explained in the next chapter, this pitch class derives from the subdominant hexachord of the medieval untransposed gamut). Since B ♭ is the only allowable flat, its harmonic significance ties it to the subdominant side of the key, and may be used as a root position chord in its own right as IV/IV and not simply as an added seventh. Harmonically, however, B ♭ would be an unlikely structural goal in C major since this pitch does not relate directly to the tonic, but to the subdominant. Therefore, our diagram indicates the pitch separated by a single line:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | D♯ |
| B ♭ | / | F – | C – | G – | d – | a – | e | ‖ | b | B |
| ♭VII | / | IV | I | V | ii | vi | iii | ‖ | vii° | V/iii |

The pitch class that follows the double line at the end of the reordered hexachord is another matter entirely. B♮ is the next fifth up from e, the terminus of the reordered C hexachord. It therefore does not exist within the C hexachord and cannot function as a goal in its own right unless the entire system is transposed. Thus a B major chord with D♯, the missing pitch of the

[7]See Burnett, "A New Theory of Hexachord Modulation in the Late 16th and Early 17th Centuries," *op. cit.*

“0” system, would imply a motion up three key signatures to a system of three sharps on A (the reason for a shift up three signatures in a tonal composition, as opposed to a modal one, is fully explained in Chapter 4). However, if b is used as a contrapuntal chord, without a major third, it can exist within the “0” system as a diminished harmony. Thus a major advantage of perceiving any given key as a reordered hexachord, is that its reordering in fifths provides the harmonic relationships that characterize the large-scale harmonic plans of most eighteenth-century music.

As the reordering shows, the primary harmonies in C major are located at the beginning of the reordered hexachord, and these are the ones most likely to be tonicized as harmonic areas closest to the prolonged tonic: C initiates the composition and G is the first goal away, whether the piece is Baroque or Classical in origin. Likewise, IV is most likely tonicized to maintain tonic stability, as in a recapitulation or a coda of a sonata-form movement, or as part of a larger movement back to the tonic after the dominant area has been explored, as in a development section. The other three triads are all minor, and they usually occur as points furthest removed from the tonic. In many major-mode eighteenth-century symphonies, for instance, either vi or iii appear only at the climax of the development section as the penultimate harmonic area before the tonic return. As one might expect, given the nature of eleven pitch-class systems, vi is more often encountered as the penultimate goal since a tonicization of iii would require the introduction of the missing pitch, effectively modulating the tonic eleven pitch-class area up three systems in the sharp direction. In the first movement of a C major string quartet in sonata form, for example, the development section ending on a E minor triad would virtually require a D♯ before the end of the retransition, thus introducing the missing pitch spelled as an augmented second above the tonic and, thus, propelling the systems three notches clockwise on the circle of fifths. Such a note would move the music into a 3♯ system. However, at the outset of the recapitulation, the appearance of the note C, most likely immediately present in the first chord of the recapitulation, would restore the tonic system since C♮ , the missing pitch of the 3♯ system spelled as a minor third above A, would move us three systems counterclockwise on the circle of fifths.

It was mentioned previously that such a procedure was very common in Baroque concertos where entire movements built toward this climax and in which the return of the tonic automatically restored the original eleven pitch class system. The return to the tonic after prolonging iii, was usually "unprepared," with no structural dominant preceding the restoration of the tonic.8 It may be concluded that the mere appearance of the tonic pitch, the corrective pitch that restored the tonic system from that of three sharps (in the case of C major), was deemed sufficient to convince the hearer that the tonic was now structurally regained.

Oddly enough, after the middle of the eighteenth century, one often finds iii (or even III♯) as the penultimate goal only in later eighteenth-century symphonies, especially in works of Joseph Haydn. Here too, Haydn sometimes omits the dominant retransition at the end of the development, and simply connects iii to the tonic recapitulation as he does in the first movement of Symphony 94 in G major. Again, the original system is always restored with the return of the tonic pitch class. What is significant, then, is that composers (at least before the later nineteenth century) generally seem reluctant to go beyond the limits of the tonic hexachord in plotting the

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[8]See reference to Corelli's opp. 1-4 above. Among countless examples of this procedure, we cite: Bach's Brandenburg Concertos no. 2 in F (first movement) and no. 4 in G (first movement), Handel's Op. 3 no. 1 in B♭/gm (first movement), and Vivaldi's Op. 4 no. 1 in B♭ (first movement).

background harmonic scheme of a movement, or even within subsections of a movement, and will generally not exceed those limits. Thus, one simply does not find B major as the penultimate goal in the development of a C major symphony, just as one would not find E major in a symphony in F. In addition to the fact that a system modulation based on the seventh scale degree would so destabilize the tonic system that it would be extremely difficult to convincingly restore it at the point of recapitulation, the seventh scale degree supports diminished harmony and is therefore incapable of acting as a localized tonic unless completely changed into a major triad, thus removing its association with the tonic on any level since it would no longer act as a leading-tone chord.

The minor mode viewed as a reordered hexachord is equally revealing. Since the minor mode is derived from the gamut of its parallel major, a reordering of C minor, for example, would follow that of an E♭ hexachord:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | F♯ |
| D♭ | / | A♭ – | E♭ – | B♭ – | f – | c – | g | ‖ | d | D |
| ♭II | / | VI | III | VII | iv | i | v | ‖ | ii° | V/V |

In the case of the minor mode, the tonal-defining chords are all minor and they are placed toward the end of the sequence. All the “strong” chords ( those that are major) appear at the beginning of the sequence. These are the harmonies that provide stability within the key, while the tonic, subdominant and dominant chords are all minor, and therefore inherently unstable.

Minor modes are unstable compared to their parallel majors due to several factors. First, there is no minor third over the fundamental in the overtone series (this fact may be the reason why there were only major hexachords in the gamut). Second, there is no major dominant, and thus no voice-leading V in the minor mode; the mode must be adjusted to create one by adding a leading tone. Third, the minor third, as an interval, has the tendency to collapse into a unison, but the major third may remain as is without further contrapuntal motion.

No wonder then that the tendency of minor mode is toward the relative major in place of the dominant! Because of its inherent instability, the minor tonic constantly seeks to move either to its parallel major (via the missing pitch of its system; here, in C minor, the missing pitch is F♯), or to simply rotate its bass to that of the tonic of its eleven pitch-class system, here three flats. Because the relative major of the mode is contained within the opening trichord of the hexachord, occupying a position of strength, its appearance neutralizes the instability of the tonic, *but it also robs the area of tension*. Thus the motion to III is inherently a stabilizing one that quite defeats the whole premise of large-scale form, that is, to create harmonic tension by moving to areas that are dissonant in relationship to that of the tonic in order to justify a return to that very tonic as a source of resolution. It would seem, then, not too much of a surprise that composers of the late eighteenth century tended to view the minor as a special case -- it was just too volatile. Why else would Haydn write only one minor-mode symphony in his late-period London group (Symphony no. 95 in C minor, discussed above), and Mozart write only two minor mode symphonies in his whole career, both in G minor?!

Notice, too, that the next fifth beyond G, the last fifth of the E ♭ hexachord, is D, supporting a diminished chord (here ii°) within the key of C minor, and, in this state, unable to function as a tonal center. However, in the minor mode, the missing pitch, here F♯, is also the leading tone of the dominant, the last pitch class of the series. Since both tonic and dominant occupy positions at the far end of the reordered hexachord, and are inherently unstable as minor chords, the missing pitch now relates to the very chords that purport to establish the key! In major mode, the missing pitch relates to a harmonic area that is clearly distantly related to the tonic. Minor mode, on the other hand, is characterized by a constant fluctuation between minor tonic and parallel major every time the missing pitch is introduced. That is, F♯ would relate to C major as its system-consonant tritone, just as an E♭ would relate to A♮ as its system-consonant tritone. Of course, in C minor, the modulatory potential of F♯ is constantly negated ("corrected") by E♭, the third of the tonic triad, the two pitch classes remaining at odds with each other over the course of the composition. Perhaps this explains why Haydn (as well as J. C. Bach and Beethoven, just to mention two) always sort to "resolve" the minor mode into its parallel major, either by the end of the first movement (the second harmonic area material being transposed from the relative major of the exposition to that of the tonic major in the recapitulation), or, at the very least, by the last movement.

It is interesting to note that the original ordering of J. S. Bach's Two-Part Inventions (which first appeared as a series of *Praeambula* in the *Klavierbüchlein* for Wilhelm Friedemann) was C (major), d (minor), e, F, G, a, b, B♭, A, g, f, E, E♭, D, and c.9 It would appear that Bach was thinking of presenting the pitch classes of the C hexachord as tonics for the first six inventions, with each invention corresponding to its proper mode within the C hexachord: the inventions in C, F, and G are in major, while the ones in d, e, and a, are in minor. Further, these six inventions share similarly constructed subjects that arpeggiate the tonic triad exclusively (the other inventions arpeggiate other chords as well), thus forming a unified set within the total work.

Throughout the eighteenth century composers and theorists alike recognized that modulation within any given key was normally confined to the roots of the first six notes of the tonic major scale.  Georg Joseph Vogler (1749-1814) in his *Tonwissenschaft und Tonsetzkunst* (Mannheim, 1776), clearly follows contemporary thought when he defines the limits of modulation within the key as follows:

> There is only one single law for modulation, but it is a general one: that there should be no going beyond a step that is one # or one ♭ removed from the key signature [...] Every piece of music [...] is named by reference to a certain key.  In order to preserve its unity, it must not modulate to a key more than one step away [...] From this is it clear that in any one key there are only *six* (emphasis mine) that can appear as principal or primary keys: e.g. in C: C major, A minor; F major, D minor; G major, E minor.10

In other words, any structural goal beyond the roots of the tonic hexachord would constitute a motion that would disrupt tonal coherence.  More specifically, a modulation beyond the tonic hexachord to a key area of more than one flat or sharp, would, in fact, lie within the tonal orbit of *another tonic hexachord*, unrelated to the original tonic key; in our terms, a hexachord from another tritone system.

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[9]Cf. Johann Sebastian Bach, *Inventionen und Sinfonien*, Georg von Dadelsen, ed. (Kassel: Bärenreiter-Verlag, 1972):iii.

[10] Georg Joseph Vogler, *Tonwissenschaft und Tonsetzkunst* (Mannheim: Kurfürstliche Hofbuchdruckerei, 1776: 70-72), cited in Ian Bent, ed. and trans., *Music Analysis in the Nineteenth Century, Volume I: Fugue, Form and Style* (Cambridge: Cambridge University Press, 1994): 143.